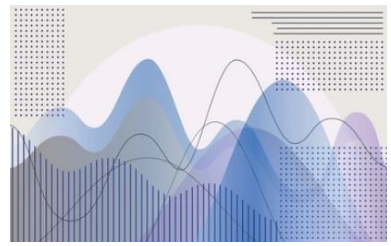
INTERCEPTS OF A PARABOLA – NON-FACTORABLE



This chapter, like an earlier one, helps you find the **intercepts** of a parabola, but the quadratic equations you will obtain will not be factorable, so another method must be utilized. You could use Completing the Square, but we'll use the Quadratic Formula.

❑ *EXAMPLES*

EXAMPLE 1: **Find the intercepts of $y = -3x^2 + 5x - 1$.**

Solution:

x-intercepts: Setting $y = 0$ turns the parabola equation into

$$\mathbf{0} = -3x^2 + 5x - 1$$

Bringing all the terms to the left side gives us the following quadratic equation in standard form:

$$3x^2 - 5x + 1 = 0$$

The left side of the equation is not factorable (give it a try!), so let's apply the Quadratic Formula, where in this case $a = 3$, $b = -5$, and $c = 1$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)}$$
$$= \frac{5 \pm \sqrt{25-12}}{6} = \frac{5 \pm \sqrt{13}}{6}$$

These two solutions are certainly correct, and we could even write our two x-intercepts like this:

$$\left(\frac{5+\sqrt{13}}{6}, 0\right) \text{ and } \left(\frac{5-\sqrt{13}}{6}, 0\right)$$

However, this form of the x-intercepts is not very useful for plotting them on a grid. It's better to use your calculator to convert the two <u>exact</u> radical answers into <u>approximate</u> decimal answers; our x-intercepts are roughly

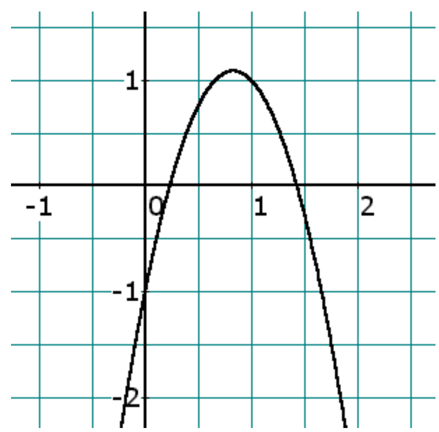
$$\boxed{(1.434,\ 0) \ \text{ and } \ (0.232,\ 0)}$$

<u>y-intercepts:</u> Setting $x = 0 \Rightarrow$

$$y = -3(\mathbf{0})^2 + 5(\mathbf{0}) - 1 = -1$$

The y-intercept is therefore

$$\boxed{(0, -1)}$$



<u>EXAMPLE 2:</u> **Find the intercepts of $y = x^2 + x + 2$.**

<u>Solution:</u> Seems easy enough, but this is a strange one.

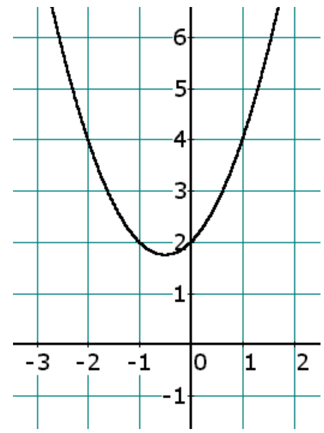
<u>x-intercepts:</u> Setting $y = 0$ yields the quadratic equation $x^2 + x + 2 = 0$. First, this quadratic won't factor, but that's okay; we have the Quadratic Formula to rescue us:

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(2)}}{2(1)} = \frac{-1 \pm \sqrt{1-8}}{2} = \frac{-1 \pm \sqrt{-7}}{2}$$

We're in trouble; the -7 in the radical sign indicates that these two solutions for x are <u>not</u> real numbers. This means that wherever these non-real (imaginary) numbers are, they are absolutely <u>not</u> on the x-axis (which is just the set of real numbers).

The conclusion of all this? This parabola has

$\boxed{\text{no } x\text{-intercept}}$



<u>y-intercepts:</u> Setting $x = 0$ gives $y = 2$, and so the y-intercept is

$\boxed{(0, 2)}$

Homework

1. Find all the **intercepts** of each parabola in exact form — no calculator, no approximations:

   a. $y = 2x^2 + 8x + 5$ b. $y = 3x^2 - 6x + 4$

   c. $y = x^2 - 13$ d. $y = 2x^2 + 9$

2. Find all the **intercepts** of each parabola, rounded to 3 digits past the point:

   a. $y = x^2 + 7x + 1$ b. $y = -2x^2 + 5x + 4$

   c. $y = 3x^2 - 6x - 2$ d. $y = 5x^2 + 3x + 1$

Solutions

1. a. $\left(\frac{-4+\sqrt{6}}{2},\ 0\right)$ $\left(\frac{-4-\sqrt{6}}{2},\ 0\right)$ (0, 5)

b. No x-intercepts (0, 4)

c. $\left(\pm\sqrt{13}, 0\right)$ (0, –13)

d. No x-intercepts (0, 9)

2. a. (–0.146, 0) (–6.854, 0) (0, 1)

b. (3.137, 0) (–0.637, 0) (0, 4)

c. (2.291, 0) (–0.291, 0) (0, –2)

d. No x-intercepts (0, 1)

"Your attitude,

not your aptitude,

will determine your altitude."

–Zig Ziglar